NON-UNIFORM FINITE DIFFERENCE METHOD FOR EUROPAN AND AMERICAN PUT OPTION USING BLACK-SCHOLES MODEL

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ABSTRACT. In this article, a numerical study for the Black-Scholes partial differential equation is introduced by using a non-uniform finite difference method to find values of European and American Put options especially at the moment of writing this option. In this case, non uniform grids can be used to solve Black-Scholes equation with better accuracy than uniform grids, because we can focus the discretization around the strike price. The stability analysis of the proposed method is given by the standard Von Neumann stability analysis. Truncation error is calculated depending on Taylor-series. Numerical test examples and comparisons have been presented for clarity.

1. INTRODUCTION

In the recent several years stock options were one of the most popular financial derivatives. There are many types of options on the market with one strike price, including American options, European options, Asian options, Exotic options, etc. Also there are many types of options with more than one strike price like Butterfly spread option. Finance and numerical analysis were the main tools to price these options. However, it was difficult to price options until 1973 when F. Black and M. S. Scholes published their Black-Scholes model [2]. The famous Black-Scholes formula is knowing nothing about what kind of option we are valuing, whether it is call option (call option is a contract that gives the right, but not the obligation, to the holder to buy the underlying asset on a particular date, or within a specified period, at a specified amount) or put option (put option is a contract that gives the right, but not the obligation, to the holder to sell the underlying asset on a particular date, or within a specified period, at a specified amount). The Black-Scholes equation with boundary and terminal conditions gives the values of an option on and before the expiration time. There are many more ways of solving parabolic partial differential equations than the explicit non-uniform method see ([4], [11], [13]). The more advanced methods are usually more complicated to program but have advantages in terms of stability [5]. This paper concentrates on
how to solve Black-Scholes partial differential model using an explicit non-uniform finite difference method to evaluate European and American Put Option. Mir in [10] used adaptive finite difference methods to simulate Butterfly option. The finite difference method is usually used with uniform mesh [9]. But, as Tavella and Randall mentioned in [12] (see also [3]), it may be useful to adapt the grid to the payoff of the option. That means, when the price of an option may be more sensitive in a precise domain, it seems legitimate to focus the mesh in that region. Such a process needs to get an axiomatic idea of the ideal mesh since one may suggest ex-ante the distribution of the discretization points of the grid. It appears that for pricing financial options, one could easily guess where the mesh has to focus on.

The plan of the paper is as follows: In Section 2, we show the basic concepts of the Black-Scholes model. In Section 3, we describe the explicit non-uniform finite difference method. In Section 4, we discuss the stability and accuracy of the presented method. Section 5, presents the computational experiments. Finally, the conclusion of the paper is given in Section 6.

2. BLACK-SCHOLES EQUATION:

We give here some notation:

- $V$: value of the option.
- $S$: price of the stock (underlying asset).
- $S^0$: the current price of the underlying asset.
- $t$: any given time.
- $r$: interest rate.
- $\mu$: the drift term of the price.
- $\sigma$: the volatility of the stock.
- $E$: the strike price of the stock (specified amount).
- $T$: the expiration time (particular date).

Suppose that we have an option whose value $V(S,t)$ depends only on $S$ and $t$, without specification whether $V$ is a call or a put option. We know the behaviour of a stock price $S$ is as the following stochastic process: $dS = \sigma S dZ + \mu S dt$. Where $dZ$ follows Wiener process, and:

- $dZ$ is a random variable.
- the mean of $dZ$ is zero.
- the variance of $dZ$ is $dt$.

Applying Itô’s lemma ([3], [13]) to the function $V(S,t)$ yields:

$$dV = \sigma S \frac{\partial V}{\partial S} dZ + (\mu S \frac{\partial V}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + \frac{\partial V}{\partial t}) dt.$$ 

This equation is difficult to solve for $V$, especially since it contains the stochastic term $dZ$. To evaluate the option $V$, we use $\Pi$ which denotes the value of an investment portfolio, $\Delta$ denotes the quantity of $S$, then: $\Pi = V - \Delta S$. The change in the portfolio is denoted as follows: $d\Pi = dV - \Delta dS$. So

$$d\Pi = \sigma S (\frac{\partial V}{\partial S} - \Delta) dZ + (\mu S \frac{\partial V}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + \frac{\partial V}{\partial t} - \mu \Delta S) dt.$$ 

If we choose

$$\Delta = \frac{\partial V}{\partial S},$$
we have
\[ d\Pi = \left( \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + \frac{\partial V}{\partial t} \right) dt. \]
Depending on the no-arbitrage principle, we can write:
\[ d\Pi = r\Pi dt \]
then:
\[ \frac{\partial V}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS \frac{\partial V}{\partial S} - rV = 0. \] (1)
Which is the Black-Scholes formula with the final condition:
\[ V_{\text{call}} = \max(S - E, 0). \]
\[ V_{\text{put}} = \max(E - S, 0). \]

3. Non-uniform Finite Difference Scheme for Black-Scholes Equation


We use a non-uniform finite difference mesh for \( S \) and a uniform finite difference mesh for \( t \) (see [1]), to obtain the full discretization finite difference formula of the Black-scholes equation (2), (see [8] to show semi-discretization formula). We shall use the following notations: \( \Delta t \) for time-step length and \( h_0, h_1, h_2, \ldots \) for price steps length, where in general \( h_i \neq h_{i+1} \), and \( \sum_i h_i = S_{\text{max}} \). Thus the coordinates of the mesh points are \( S_i = h_{i-1} + S_{i-1} \) and \( t_j = j\Delta t \). Let the values of the solution \( V(S,t) \) on these grid points are approximated by \( V(S_i, t_j) = V_{i,j} \).

3.2. Discretisation Scheme for the Black-Scholes Equation.

Taylor-series approximations for 1\textsuperscript{st} and 2\textsuperscript{nd} order derivatives of \( V(S,t) \) are known as follows:
\[ V(S_{i+1}, t_j) = V(S_i, t_j) + h_i \frac{\partial V(S_i, t_j)}{\partial S} + \frac{h_i^2}{2} \frac{\partial^2 V(S_i, t_j)}{\partial S^2} + O(h_i^3). \]
\[
V(S_{i-1}, t_j) = V(S_i, t_j) - h_{i-1} \frac{\partial V(S_i, t_j)}{\partial S} + \frac{h_{i-1}^2}{2} \frac{\partial^2 V(S_i, t_j)}{\partial S^2} - O(h_{i-1}^3).
\]

We can find from these relations: Forward time difference:
\[
\frac{\partial V(S_i, t_j)}{\partial t} = \frac{V(S_i, t_{j+1}) - V(S_i, t_j)}{\Delta t} + O(\Delta t) = D^+_t V(S_i, t_j) + O(\Delta t). \tag{3}
\]

Backward time difference:
\[
\frac{\partial V(S_i, t_j)}{\partial t} = \frac{V(S_i, t_j) - V(S_i, t_{j-1})}{\Delta t} + O(\Delta t) = D^-_t V(S_i, t_j) + O(\Delta t). \tag{4}
\]

Forward space difference:
\[
\frac{\partial V(S_i, t_j)}{\partial S} = \frac{V(S_{i+1}, t_j) - V(S_i, t_j)}{h_i} + O(h_i) = D^+_S V(S_i, t_j) + O(h_i). \tag{5}
\]

Backward space difference:
\[
\frac{\partial V(S_i, t_j)}{\partial S} = \frac{V(S_i, t_j) - V(S_{i-1}, t_j)}{h_{i-1}} + O(h_i) = D^-_S V(S_i, t_j) + O(h_i). \tag{6}
\]

Central space difference:
\[
\frac{\partial V(S_i, t_j)}{\partial S} = \frac{V(S_{i+1}, t_j) - V(S_{i-1}, t_j)}{h_i + h_{i-1}} + O(h_i + h_{i-1}) = D_S V(S_i, t_j) + O(h_i + h_{i-1}). \tag{7}
\]

Second space difference:
\[
\frac{\partial^2 V(S_i, t_j)}{\partial S^2} = \frac{D^+_S V(S_i, t_j) - D^-_S V(S_i, t_j)}{(h_i + h_{i-1})/2} + O(h_i + h_{i-1}) = D^2_S V(S_i, t_j) + O(h_i + h_{i-1}). \tag{8}
\]

Now, to obtain the explicit non-uniform finite difference scheme of the Black-Scholes equation \[2\], we are going to replace the partial derivatives in this equation by their approximations in \[3\], \[7\] and \[8\].

\[
D^-_t V(S_i, t_j) - \frac{1}{2} \sigma^2 S_i^2 D^2_S V(S_i, t_j) - r S_i D_S V(S_i, t_j) + r V_i, j = T_i, j. \tag{9}
\]

Where \( T_{i,j} \) is the resulting truncation error. The standard difference formula is given by:

\[
\frac{V_{i,j+1} - V_{i,j}}{\Delta t} - \frac{1}{2} \sigma^2 S_i^2 \left( \frac{2}{h_i(h_i + h_{i-1})} V_{i+1,j} - \frac{2}{h_{i-1}(h_i + h_{i-1})} V_{i-1,j} \right) - \frac{2}{h_i h_{i-1}} V_{i,j} + \frac{2}{h_{i-1}(h_i + h_{i-1})} V_{i-1,j} - r S_i \left( \frac{V_{i+1,j} - V_{i-1,j}}{h_i + h_{i-1}} \right) + r V_{i,j} = 0.
\]

Rearrange this difference equation using \( S_i = h_0 + h_1 + ... + h_{i-1} \), we find:
\[
V_{i,j+1} = \alpha_i V_{i-1,j} + \beta_i V_{i,j} + \gamma_i V_{i+1,j}. \tag{10}
\]

Where the coefficient:
\[
\alpha_i = \left( \sigma^2 \frac{S_i^2}{h_{i-1}(h_i + h_{i-1})} - r \frac{S_i}{h_i + h_{i-1}} \right) \Delta t.
\]

\[
\beta_i = 1 - \left( \sigma^2 \frac{S_i^2}{h_i(h_i + h_{i-1})} + r \right) \Delta t.
\]
\[ \gamma_i = (\sigma^2 \frac{S_i^2}{h_i(h_i + h_{i-1})} + r \frac{S_i}{(h_i + h_{i-1})}) \Delta t. \]

4. Stability Analysis and Truncating Error

4.1. Stability Analysis.

In this section, we use the Von Neumann method to study the stability analysis of the Black-Scholes equation. Stability criteria arises from the balancing of the time dependency and diffusion terms in this equation \[9\], so we have the following theorem:

**Theorem 1.** The explicit finite-difference scheme \[10\] for the Black-Scholes equation with the coefficient:

\[ \alpha_i = \sigma^2 \frac{S_i^2}{h_{i-1}(h_i + h_{i-1})} \Delta t, \]

\[ \beta_i = 1 - \sigma^2 \frac{S_i^2}{h_i \cdot h_{i-1}} \Delta t, \]

\[ \gamma_i = \sigma^2 \frac{S_i^2}{h_i(h_i + h_{i-1})} \Delta t. \]

is conditionally stable if

\[ \Delta t \leq \frac{2A}{A^2 + B^2} : \Delta t > 0. \]

where

\[ A = \sigma^2 S_i^2 \left[ \frac{1}{h_i \cdot h_{i-1}} - \frac{1}{h_i(h_i + h_{i-1})} \cos(qh_i) - \frac{1}{h_{i-1}(h_i + h_{i-1})} \cos(qh_{i-1}) \right], \]

\[ B = \sigma^2 S_i^2 \left[ \frac{1}{h_i(h_i + h_{i-1})} \sin(qh_i) - \frac{1}{h_{i-1}(h_i + h_{i-1})} \sin(qh_{i-1}) \right]. \]

**Proof.** Let us analyze the stability of \[10\] by substituting in a separated solution

\[ V_{i,j} = \zeta_j e^{\sqrt{q} S_i} = \zeta_j e^{\sqrt{q}(h_0 + h_1 + \ldots + h_{i-1})} : q \text{ is a real number}. \]

Inserting this expression in \[10\] we get:

\[ \zeta_{j+1} e^{\sqrt{q} S_i} = \sigma^2 \frac{S_i^2}{h_{i-1}(h_i + h_{i-1})} \Delta t \zeta_j e^{\sqrt{q} S_i} + (1 - \sigma^2 \frac{S_i^2}{h_i \cdot h_{i-1}} \Delta t) \zeta_j e^{\sqrt{q} S_i}, \]

\[ + \sigma^2 \frac{S_i^2}{h_i(h_i + h_{i-1})} \Delta t \zeta_j e^{\sqrt{q} S_i}. \]

(11)

Dividing (11) by \( e^{\sqrt{q} S_i} \), we get:

\[ \zeta_{j+1} = \sigma^2 \frac{S_i^2}{h_{i-1}(h_i + h_{i-1})} \Delta t \zeta_j e^{-\sqrt{q} h_{i-1}} + (1 - \sigma^2 \frac{S_i^2}{h_i \cdot h_{i-1}} \Delta t) \zeta_j + \sigma^2 \frac{S_i^2}{h_i(h_i + h_{i-1})} \Delta t \zeta_j e^{-\sqrt{q} h_i}. \]

\[ \zeta_{j+1} = \zeta_j \left[ \sigma^2 \Delta t \left( \frac{S_i^2}{h_{i-1}(h_i + h_{i-1})} e^{-\sqrt{q} h_{i-1}} + \frac{S_i^2}{h_i(h_i + h_{i-1})} e^{-\sqrt{q} h_i} \right) + (1 - \sigma^2 \frac{S_i^2}{h_i \cdot h_{i-1}} \Delta t) \right]. \]

\[ \zeta_{j+1} = \zeta_j \left[ \sigma^2 \Delta t S_i^2 \frac{1}{h_i(h_i + h_{i-1})} e^{-\sqrt{q} h_{i-1}} + \frac{1}{h_i(h_i + h_{i-1})} e^{-\sqrt{q} h_i} - \frac{1}{h_i \cdot h_{i-1}} \right] + 1]. \]
In the Von Neumann method, the stability analysis is carried out using the amplification factor \( \eta \) defined by: \( \zeta_{j+1} = \eta \zeta_j \). Sure, \( \eta \) is independent on \( j \). Then, we have the following formula of \( \eta \):

\[
\eta = [\sigma^2 \Delta t S_i] \left( \frac{1}{h_{i-1}(h_i + h_{i-1})} e^{-\sqrt{\eta} q h_{i-1}} + \frac{1}{h_i(h_i + h_{i-1})} e^{\sqrt{\eta} q h_i} - \frac{1}{h_i \cdot h_{i-1}} \right) + 1].
\]

The model will be stable as long as: \( |\eta| \leq 1 \), i.e.:

\[
|\sigma^2 \Delta t S_i| \left( \frac{1}{h_{i-1}(h_i + h_{i-1})} e^{-\sqrt{\eta} q h_{i-1}} + \frac{1}{h_i(h_i + h_{i-1})} e^{\sqrt{\eta} q h_i} - \frac{1}{h_i \cdot h_{i-1}} \right) + 1] \leq 1.
\]

Using the known Euler’s formula \( e^{\sqrt{-1} \theta} = \cos \theta + \sqrt{-1} \sin \theta \), we have:

\[
|\sigma^2 \Delta t S_i| \left( \frac{1}{h_{i-1}(h_i + h_{i-1})} (\cos(q h_{i-1}) - \sqrt{-1} \sin(q h_{i-1}))
+ \frac{1}{h_i(h_i + h_{i-1})} (\cos(q h_i) + \sqrt{-1} \sin(q h_i)) - \frac{1}{h_i \cdot h_{i-1}} \right) + 1] \leq 1.
\]

\[
|1 - \sigma^2 \Delta t S_i| \left[ \frac{1}{h_i \cdot h_{i-1}} - \frac{1}{h_{i-1}(h_i + h_{i-1})} \cos(q h_{i-1}) - \frac{1}{h_i(h_i + h_{i-1})} \cos(q h_i)
+ \sqrt{-1} \cdot \sigma^2 \Delta t S_i \right] \left[ \frac{-1}{h_{i-1}(h_i + h_{i-1})} \sin(q h_{i-1}) + \frac{1}{h_i(h_i + h_{j-1})} \sin(q h_i) \right] \leq 1.
\]

Let

\[
A = \sigma^2 S_i \left[ \frac{1}{h_i \cdot h_{i-1}} - \frac{1}{h_{i-1}(h_i + h_{i-1})} \cos(q h_{i-1}) - \frac{1}{h_i(h_i + h_{i-1})} \cos(q h_i) \right]
\]

\[
B = \sigma^2 S_i \left[ \frac{1}{h_i(h_i + h_{i-1})} \sin(q h_i) - \frac{1}{h_{i-1}(h_i + h_{i-1})} \sin(q h_{i-1}) \right]
\]

Then we have:

\[
|1 - \Delta t A + \sqrt{-1} \Delta t B| \leq 1.
\]

That means:

\[
(1 - \Delta t A)^2 + (\Delta t B)^2 \leq 1.
\]

i.e.,

\[
1 + \Delta t^2 A^2 - 2 \Delta t A + \Delta t^2 B^2 \leq 1.
\]

\[
\Delta t \leq \frac{2A}{A^2 + B^2}, \quad \Delta t > 0.
\]
4.2. Truncation Error.

**Theorem 2.** The truncation error of non-uniform finite difference method for the Black-Scholes equation is:

\[ T_{i,j} = O(\Delta t) + \sigma^2 S_i^2 O(h) + 2rS_i O(h). \]

\[ h = \max_{i=0,1,2,...,M} \{h_i\}. \]

**Proof.** From the definition of truncating error given by Eq. (9), we have:

\[ T_{i,j} = D_t V(S_i, t_j) - \frac{1}{2} \sigma^2 S_i^2 D_S^2 V(S_i, t_j) - rS_i D_S V(S_i, t_j) + rV_{i,j}. \]

If we put

\[ h = \max_{i=1,2,...,M} \{h_i, h_{i-1}\}, \]

we finally obtain:

\[ T_{i,j} = O(\Delta t) + \sigma^2 S_i^2 O(h_i + h_{i-1}) + 2rS_i O(h_i + h_{i-1}). \]

5. Numerical Results

In this section, we will use particular non-uniform mesh (3, 7) to find the non-uniform finite difference approximations of the put options values at \( t = 0 \) when \( S = S^0 \). The results are compared with the uniform finite difference approximations.

The non-uniform grid, which used, is as follows:

\[ S_i = E + \alpha \sinh(c_1 \cdot i/M + c_2 \cdot (1 - i/M)), \: i = 0, 1, ..., M. \]

\[ c_1 = \sinh^{-1}((S_{\text{max}} - E)/\alpha), \]

\[ c_2 = \sinh^{-1}((S_0 - E)/\alpha). \]

Figure 1 shows the non-uniform mesh when we use this grid on \([0, 100]\) where \( E = 50 \), and \( \alpha = 4, 20, 50 \).
Example 1: If the parameters of Black-Scholes equation (1) are:

\[ S^0 = E = 40, \ r = 0.1, \ \sigma = 0.3, \ T = 1, \ S_{\text{max}} = 90. \]

We will find the value of European put option with final condition:

\[ V(S,T) = \text{Max}(E - S, 0). \]

And boundary conditions:

\[ V(0,t) = Ee^{-r(T-t)}, \]
\[ V(S_{\text{max}},t) = 0. \]

The exact solution of Black-Scholes in this case: see (7), (13)

\[ V_{\text{European}} = Ee^{-r(T-t)} \cdot N(-d_2) - S \cdot N(-d_1). \]

such that:

\[ d_1 = (\ln(S/E) + (r + 0.5\sigma^2)(T-t))/(\sigma\sqrt{T-t}), \]
\[ d_2 = d_1 - \sigma\sqrt{T-t}. \]

Where \( N(.) \) is the normal distribution function, given by

\[ N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}y^2} dy. \]

The numerical studies are given as follows: if we take \( \alpha = 4, \ M = 20, \) (number of price nodes) and different values of \( N, \) (number of time nodes). Table 1 shows the three solutions, numerical approximation non-uniform finite difference, numerical approximation uniform finite difference and analytical solution, and absolute errors, at time \( t = 0 \) and \( S = S^0, \) between the exact solution and the two numerical solutions. Figure 2 shows a comparison between the behavior of the exact solution and the non-uniform approximate solution when \( M=25 \) and \( N=50 \) at \( t=0, \) and figure 3 shows a comparison between the behavior of the exact solution and the uniform approximate solution when \( M=25 \) and \( N=50 \) at \( t=0. \)

Example 2: If the parameters of the equation (1) are:

\[ S^0 = 1.25, \ E = 1.2, \ r = 0.05, \ \sigma = 0.25, \ T = 2, \ S_{\text{max}} = 2.5. \]
Table 1. Example 1 solutions, numerical approximation non-uniform finite difference, uniform finite difference, and analytical solution, and absolute errors, at time $t=0$ and $S=S^0$

<table>
<thead>
<tr>
<th>$M$</th>
<th>$N$</th>
<th>$V_{\text{nonuniform}}$</th>
<th>$V_{\text{uniform}}$</th>
<th>$V_{\text{exact}}$</th>
<th>$\text{error}_{\text{nonuniform}}$</th>
<th>$\text{error}_{\text{uniform}}$</th>
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<td>2.8647</td>
<td>2.8872</td>
<td>0.0012</td>
<td>0.0224</td>
</tr>
</tbody>
</table>

We will find the value of American put option with final condition:

$$V(S,T) = \max(E - S, 0).$$

Boundary conditions:

$$V(0,t) = Ee^{-r(T-t)}, \quad V(S_{\text{max}},t) = 0.$$ and

$$V_{\text{American}}(S,t) = \max(E - S, V_{\text{European}}(S,t)).$$

But with no exact solution [13].

Remark: In American option the holder can exercise an option during time interval, before its expiry date, then the value of the option $V_{\text{American}}$ can not be less than the payoff= $\max(E - S, 0)$ during that time period, so $V_{\text{American}}(S,t) \geq \max(E - S, 0)$ and $V_{\text{American}}(S,T) \geq V_{\text{European}}(S,t)$ (9), (13). The numerical studies are given as follows: if we take $\alpha = 2$, $M = 10$, and different values of $N$. Table 2 shows the two solutions, numerical approximation non-uniform finite difference, numerical approximation uniform finite difference for European put option and American put option, at time $t = 0$ and $S = S^0$. Figure 4 shows the behavior of the non-uniform approximation of values of American put option when M=25 and N=50 at $t=0$, and figure 5 shows the behavior of the uniform approximation of values of American put option when M=25 and N=50 at $t=0$.

We can notice from table 2 that the values of the American put option is greater than the European put option when using both uniform and this non-uniform grid.
Table 2. Numerical approximation non-uniform finite difference, uniform finite difference for example 2, European and American put option, at time $t=0$ and $S = S^0$

<table>
<thead>
<tr>
<th>$M$</th>
<th>$N$</th>
<th>$V_{\text{nonuniform}}$</th>
<th>$V_{\text{uniform}}$</th>
<th>$V_{\text{nonuniform}}$</th>
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</tr>
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</table>

And because this non-uniform method is more accurate than uniform in our case so the values of the American put option which was calculated using this non-uniform grid is closer to the actual value than which calculated using uniform grid.

6. Conclusion

This paper presented a class of numerical methods for finding the value of options using Black-scholes formula. This class of methods is a non-uniform finite difference method. Special attention is given to study the stability and consistency of proposed methods. To execute this aim we have resorted to the kind of John Von Neumann stability analysis. The presented stability criterion of the Black-Scholes equation depends strongly on the values of $h_i$ and the volatility of the stock. Two numerical test examples are presented. We can conclude from the comparison between the uniform and the non-uniform finite difference that, for some kind of grid, the non-uniform leads to faster convergence and more accurate results than uniform finite difference. We can say that, the numerical non-uniform approximation of values of American option are in excellent agreement with the real values.

References


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