COMMON FIXED POINT THEOREM IN MENGER SPACE USING (CLRg) PROPERTY

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ABSTRACT. The object of this paper is to establish a common fixed point theorem for semi-compatible pair of self maps by using CLRg Property in Menger space.

1. INTRODUCTION

There have been a number of generalizations of metric space. One such generalization is Menger space initiated by Menger [2]. It is a probabilistic generalization in which we assign to any two points \( x \) and \( y \), a distribution function \( F_{x,y} \). Schweizer and Sklar [6] studied this concept and gave some fundamental results on this space. Sehgal and Bharucha-Reid [7] obtained Banach contraction principal in a complete Menger space, which is a milestone in developing fixed point theory in Menger space. Sessa [8] initiated the tradition of improving comutativity in fixed point theorems by introducing the notion of weakly commuting maps in metric spaces. Jungck [1] soon enlarged this concept to compatible maps. The notion of compatible mapping in a Menger space has been introduced by Mishra [3]. Pant [4] introduced the notion of reciprocal continuity of mappings in metric spaces. Popa [5] proved theorem for weakly compatible non-continuous mapping using implicit relation. Singh and Jain [9] have been introduced semi-compatible, compatible and weak compatible maps in Menger space.

B. Singh et. al. [10] introduced the notion of semi compatible maps in fuzzy metric space. In 2011, Sintunayarat and Kuman [11] introduced the concept of common limit in the range property. Chouhan et. al. [12] utilize the notion of common limit range property to prove fixed point theorems for weakly compatible mapping in fuzzy metric space.

In 2012, Jain et al. [13] extended the concept of CLRg property in the coupled case and also established a common fixed point theorem for weakly compatible maps.
mappings in fuzzy metric spaces. Most recently, Hierro and Sintunavarat \[14\] generalized the results in \[13\] by using the generalized contractive conditions and the CLRg property in fuzzy metric spaces.

2. Preliminaries

**Definition 2.1** A mapping $F : \mathbb{R} \rightarrow \mathbb{R}^+$ is called a distribution if it is non-decreasing left continuous with $\inf\{F(t) : t \in \mathbb{R}\} = 0$ and $\sup\{F(t) : t \in \mathbb{R}\} = 1$. We shall denote by $L$ the set of all distribution functions while $H$ will always denote the specific distribution function defined by

$$H(x) = \begin{cases} 0, & x \leq 0 \\ 1, & x > 0. \end{cases}$$

**Definition 2.2** A Probabilistic metric space (PM-space) is an ordered pair $(X, F)$, where $X$ is an abstract set of elements and $F : X \times X \rightarrow L$, defined by $(p, q) \rightarrow F_{p,q}$, where $L$ is the set of all distribution functions i.e. $L = \{F_{p,q} / p, q \in X\}$, if the functions $F_{p,q}$ satisfy.

(a) $F_{p,q}(x) = 1$, for all $x > 0$, if and only if $p = q$;
(b) $F_{p,q}(0) = 0$;
(c) $F_{p,q} = F_{q,p}$;
(d) If $F_{p,q}(x) = 1$ and $F_{q,r}(y) = 1$ then $F_{p,r}(x+y) = 1$.

**Definition 2.3** A mapping $t : [0, 1] \times [0, 1] \rightarrow [0, 1]$ is called a $t$-norm if

(a) $t(a, 1) = a$;
(b) $t(a, b) = t(b, a)$;
(c) $t(c, d) \geq t(a, b)$ for $c \geq a, d \geq b$;
(d) $t(t(a, b), c) = t(a, t(b, c))$, for all $a, b, c, d \in [0, 1]$.

**Definition 2.4** A Menger space is a triplet $(X, F, t)$ where $(X, F)$ is PM-space and $t$ is a $t$-norm such that for all $x, y \geq 0$

$$F_{p,r}(x + y) \geq t(F_{p,q}(x), F_{q,r}(y)).$$

Schweizer and Sklar \[6\] proved that if $(X, F, t)$ is a Menger space with $\sup_{0 < x < 1} t(x, x) = 1$, then $(X, F, t)$ is a Hausdorff topological space in the topology induced by the family of $(\varepsilon, \lambda)$-neighborhoods, $\{U_p(\varepsilon, \lambda) : p \in X, \varepsilon > 0, \lambda > 0\}$, where $U_p(\varepsilon, \lambda) = \{x \in X : F_{x,p}(\varepsilon) > 1 - \lambda\}$.

**Definition 2.5** Let $(X, F, t)$ be a Menger space with $\sup_{0 < x < 1} t(x, x) = 1$. A sequence $\{p_n\}$ in $X$ is said to converge to a point $p$ in $X$ (written as $p_n \rightarrow p$) if for every $\varepsilon > 0$ and $\lambda > 0$, there exists an integer $M(\varepsilon, \lambda)$ such that $F_{p_n, p}(\varepsilon) > 1 - \lambda$, $\forall n \geq M(\varepsilon, \lambda)$. Further, the sequence is said to be a cauchy sequence if for each $\varepsilon > 0$ and $\lambda > 0$, there exists an integer $M(\varepsilon, \lambda)$ such that $F_{p_m, p_n}(\varepsilon) > 1 - \lambda$, $\forall n, m \geq M(\varepsilon, \lambda)$. A Menger space $(X, F, t)$ is said to be complete if every cauchy sequence in it converges to a point of it.

A complete metric space can be treated as a complete menger space in the following way.
Proposition 2.6 If \((X, d)\) is a metric space then the metric \(d\) induces a mapping \(X \times X \rightarrow L\), defined by \(F_{p,q}(x) = H(x - d(p, q))\), \(\forall p, q \in X\) and \(x \in R\). Further, if \(t : [0, 1] \times [0, 1] \rightarrow [0, 1]\) is defined by \(t(a, b) = \min\{a, b\}\), then \((X, F, t)\) is a Menger space. It is complete if \((X, d)\) is complete. Then space \((X, F, t)\) so obtained is called the induced Menger space.

Proposition 2.7 In a Menger space \((X, F, t)\), if \(t(x, x) \geq x, \ \forall x \in [0, 1]\) then \(t(a, b) = \min\{a, b\}, \ \forall a, b \in [0, 1]\).

Definition 2.8 Self mappings \(A\) and \(S\) of a Menger space \((X, F, t)\) are said to be weak compatible if they commute at their coincidence points i.e. \(Ax = Sx\) for \(x \in X\) implies \(ASx = SAx\).

Definition 2.9 Self mappings \(A\) and \(S\) of a Menger space \((X, F, t)\) are called compatible if \(F_{AS, p, PA}(x) \rightarrow 1, \ \forall x > 0\) whenever \(\{p_n\}\) is a sequence in \(X\) such that \(A_{p_n}, S_{p_n} \rightarrow u, \ \forall u \in X, \ \text{as} \ n \rightarrow \infty\).

Definition 2.10 Self mappings \(A\) and \(S\) of a Menger space \((X, F, t)\) are called semi-compatible if \(F_{AS, p, S, p}(x) \rightarrow 1, \ \forall x > 0\) whenever \(\{p_n\}\) is a sequence in \(X\) such that \(A_{p_n}, S_{p_n} \rightarrow u, \ \forall u \in X, \ \text{as} \ n \rightarrow \infty\).

Proposition 2.11 If self mappings \(A\) and \(S\) of a Menger space \((X, F, t)\) are semi-compatible then they are weak compatible.

Proposition 2.12 Let \(S\) and \(T\) be two self maps on a Menger space \((X, F, t)\) with \(t(a, a) \geq a, \ \forall a \in [0, 1]\) of which \(T\) is continuous. Then \((S, T)\) is semi-compatible if and only if \((S, T)\) is compatible.

Lemma 2.13 Let \(\{p_n\}\) be a sequence in a Menger space \((X, F, t)\) with continuous \(t\)-norm \(t(x, x) \geq x, \ \forall x \in [0, 1]\). If \(k \in (0, 1)\) such that for all \(x > 0\) and \(n \in N\), \(F_{pm, pm+1}(kx) \geq F_{pm-1, pm}(x)\). Then \(\{p_n\}\) is a Cauchy sequence in \(X\).

Definition 2.14 Let \((X, F, t)\) be a menger space with continuous \(t\)-norm \(t(x, x) \geq x, \ \forall x \in [0, 1]\). Then two mappings \(f, g : X \rightarrow X\) are said to have the CLR property if there exist a sequence \(\{x_n\}\) in \(X\) and a point \(z\) in \(X\) such that

\[
\lim_{n \to \infty} f x_n = \lim_{n \to \infty} g x_n = g z.
\]

Definition 2.15 Two pairs \((A, S)\) and \((B, T)\) of self mappings of a menger space \((X, F, t)\) are said to satisfy the \((CLR_{ST})\) property if there exist two sequence \(\{x_n\}\) and \(\{y_n\}\) in \(X\) such that

\[
\lim_{n \to \infty} A x_n = \lim_{n \to \infty} S x_n = \lim_{n \to \infty} B y_n = \lim_{n \to \infty} T y_n = S z, \ \text{for some} \ z \in S(X) \ \text{and} \ z \in T(X).
\]

Definition 2.16 Two pairs \((A, S)\) and \((B, T)\) of self mappings of a menger space \((X, F, t)\) are said to share \(CLRg\) of \(S\) property if there exist two sequence \(\{x_n\}\) and
\{y_n\} in X such that
\[
\lim_{n \to \infty} Ax_n = \lim_{n \to \infty} Sx_n = \lim_{n \to \infty} By_n = \lim_{n \to \infty} Ty_n = Sz, \text{ for some } z \in X.
\]

**Example 2.17** Let \( X = \{0, \infty\} \) be the usual metric space. Define \( g, h : X \to X \) by \( gx = x + 3 \) and \( gx = 4x \), for all \( x \in X \). We consider the sequence \( \{x_n\} = \{1 + 1/n\} \). Since, \( \lim_{n \to \infty} gx_n = \lim_{n \to \infty} hx_n = 4 = h(1) \in X \). Therefore \( g \) and \( h \) satisfy the (CLRg) property.

**Definition 2.18** We will apply an implicit relation as, Let \( \Phi \) be set of all real continuous functions \( \phi : (R^+)^4 \to R \), nondecreasing in first argument and satisfying the following conditions:

(i) For \( u, v \geq 0, \phi(u, v, v, u) \geq 0 \) or \( \phi(u, v, u, v) \geq 0 \) imply \( u \geq v \).
(ii) \( \phi(u, u, 1, 1) \geq 0 \) implies \( u \geq 1 \).

**Example 2.19** Define \( \phi(t_1, t_2, t_3, t_4) = 15t_1 - 13t_2 + 5t_3 - 7t_4 \). Then \( \phi \in \Phi \).

### 3. Main Result

In the following theorem we replace the continuity condition by using (CLRg) property.

**Theorem 3.1** Let \( A, B, S \) and \( T \) be self mapping on a complete menger space \((X, F, t)\), satisfying

(a) \( A(X) \subseteq T(X) \), \( B(X) \subseteq S(X) \),
(b) \((B, T)\) is semi compatible,
(c) For some \( \phi \in \Phi \), there exists \( k \in (0, 1) \) such that for all \( x, y \in X \) and \( t > 0 \),

\[
\phi(F_{Ax,By}(kt), F_{Sx,Ty}(t), F_{Ax,Sx}(t), F_{By,Ty}(kt)) \geq 0 \quad (1)
\]
\[
\phi(F_{Ax,By}(kt), F_{Sx,Ty}(t), F_{Ax,Sx}(kt), F_{By,Ty}(t)) \geq 0 \quad (2)
\]

If the pair \((A, S)\) and \((B, T)\) share the common limit in the range of \( S \) property, then \( A, B, S \) and \( T \) have a unique common fixed point.

**Proof.** Let \( x_0 \) be an arbitrary point in \( X \). Since \( A(X) \subseteq T(X) \) and \( B(X) \subseteq S(X) \), there exist \( x_1, x_2 \in X \) such that \( Ax_0 = Tx_1 \) and \( Bx_1 = Sx_2 \). Inductively, we construct the sequences \( \{y_n\} \) and \( \{x_n\} \) in \( X \) such that

\[
y_{2n+1} = Ax_{2n} = Tx_{2n+1}, \quad y_{2n+2} = Bx_{2n+1} = Sx_{2n+2}
\]

for \( n = 0, 1, 2, \ldots \). Now putting in (1) \( x = x_{2n}, y = x_{2n+1} \), we obtain

\[
\phi(F_{Ax_{2n},Bx_{2n+1}}(kt), F_{Sx_{2n},Tx_{2n+1}}(t), F_{Ax_{2n},Sx_{2n}}(t), F_{Bx_{2n+1},Tx_{2n+1}}(kt)) \geq 0
\]

that is

\[
\phi(F_{y_{2n+1},y_{2n+2}}(kt), F_{y_{2n},y_{2n+1}}(t), F_{y_{2n+1},y_{2n}}(t), F_{y_{2n+2},y_{2n+1}}(kt)) \geq 0
\]

Using (i), we get

\[
F_{y_{2n+2},y_{2n+1}}(kt) \geq F_{y_{2n+1},y_{2n}}(t) \quad (3)
\]
Analogously, putting \( x = x_{2n+2}, \ y = x_{2n+1} \) in (2), we have
\[
\phi\left( F_{Ax_{2n}, Bx_{2n+1}}(kt), F_{x_{2n+2}, Tx_{2n+1}}(t), F_{Ax_{2n+2}, Sx_{2n+2}}(kt), F_{Bx_{2n+1}, Tx_{2n+1}}(t) \right) \geq 0
\]
\[
\phi\left( F_{y_{2n+3}, y_{2n+2}}(kt), F_{y_{2n+2}, y_{2n+1}}(t), F_{y_{2n+3}, y_{2n+3}}(kt), F_{y_{2n+2}, y_{2n+1}}(t) \right) \geq 0
\]
Using (i), we get
\[
F_{y_{2n+3}, y_{2n+2}}(kt) \geq F_{y_{2n+1}, y_{2n+2}}(t) \tag{4}
\]
Thus, from (3) and (4), for \( n \) and \( t \), we have
\[
F_{y_{n+3}, y_{n+2}}(kt) \geq F_{y_{n+1}, y_{n+2}}(t)
\]
Hence, by Lemma 2.13, \( \{y_n\} \) is a Cauchy sequence in \( X \), which is complete. Therefore, \( \{y_n\} \) converges to \( z \) in \( X \). That is \( \{Ax_{2n}\}, \{Tx_{2n+1}\}, \{Bx_{2n+1}\} \) and \( \{Sx_{2n}\} \) also converges to \( z \) in \( X \).

Since the pair \( (A, S) \) and \( (B, T) \) share the common limit in the range of \( S \) property, then there exist two sequences \( \{x_n\} \) and \( \{y_n\} \) in \( X \) such that
\[
\lim_{n \to \infty} Ax_n = \lim_{n \to \infty} Sx_n = \lim_{n \to \infty} By_n = \lim_{n \to \infty} Ty_n = Sz, \text{ for some } z \in X
\]
First we prove that \( Az = Sz \)

By (1), putting \( x = z \) and \( y = y_n \), we get
\[
\phi\left( F_{Az, By_n}(kt), F_{Sx, Ty_n}(t), F_{Az, Sz}(t), F_{By_n, Ty_n}(kt) \right) \geq 0
\]
Taking limit \( n \to \infty \), we get
\[
\phi\left( F_{Az, Sz}(kt), F_{Sx, Sz}(t), F_{Az, Sz}(t), F_{Sx, Sz}(kt) \right) \geq 0
\]
As \( \phi \) is non-decreasing in first argument, we have
\[
\phi\left( F_{Az, Sz}(t), 1, F_{Az, Sz}(t), 1 \right) \geq 0
\]

using (ii), we have
\[
F_{Az, Sz}(t) \geq 1 \text{ for all } t > 0
\]
which gives \( F_{Az, Sz}(t) = 1 \), that is \( Az = Sz \) \tag{5}

Since, \( A(X) \subseteq T(X) \), therefore there exist \( u \in X \), such that \( Az = Tu \) \tag{6}

Again by inequality (1), putting \( x = z \) and \( y = u \), we get
\[
\phi\left( F_{Az, Bu}(kt), F_{Sx, Tu}(t), F_{Az, Sz}(t), F_{Bu, Tu}(kt) \right) \geq 0
\]
using (5) and (6), we get
\[
\phi\left( F_{Tu, Bu}(kt), F_{Az, Az}(t), F_{Sz, Sz}(t), F_{Bu, Tu}(kt) \right) \geq 0
\]
\[
\phi\left( F_{Tu, Bu}(kt), 1, 1, F_{Bu, Tu}(kt) \right) \geq 0
\]
using (i), we have \( F_{Tu, Bu}(kt) \geq 1 \) for all \( t > 0 \),
which gives \( F_{Tu, Bu}(kt) = 1 \). Thus \( Tu = Bu \) \tag{7}
Thus from (5), (6), (7), we get \( Az = Sz = Tu = Bu \) \tag{8}
By inequality (1), putting \( x = z \) and \( y = x_{2n+1} \),

\[
\phi \left( F_{A_{z}, B_{x_{2n+1}}}(kt), F_{S_{z}, T_{x_{2n+1}}}(t), F_{A_{z}, S_{z}}(t), F_{B_{x_{2n+1}}, T_{x_{2n+1}}}(kt) \right) \geq 0
\]

taking limit \( n \to \infty \), using (i) we get

\[
\phi \left( F_{A_{z}, z}(kt), F_{S_{z}, z}(t), F_{A_{z}, S_{z}}(t), F_{z, z}(kt) \right) \geq 0
\]

\[
\phi \left( F_{A_{z}, z}(kt), F_{A_{z}, z}(t), F_{A_{z}, A_{z}}(t), F_{z, z}(kt) \right) \geq 0
\]

\[
\phi \left( F_{A_{z}, z}(kt), F_{A_{z}, z}(t), 1, 1 \right) \geq 0
\]
as \( \phi \) is non-decreasing in first argument, we have

\[
\phi \left( F_{A_{z}, z}(t), F_{A_{z}, z}(t), 1, 1 \right) \geq 0
\]

using (ii), we have \( F_{A_{z}, z}(t) \geq 1 \) for all \( t > 0 \), which gives \( F_{A_{z}, z}(t) = 1 \). Thus \( A_{z} = z \).

Therefore from (8), we get \( z = T_{u} = Bu \)

Now Semicompatibility of \((B, T)\) gives \( BT_{y_{2n+1}} \to T_{z} \), i.e. \( B_{z} = T_{z} \) Now putting \( x = z \) and \( y = z \) in inequality (1), we get

\[
\phi \left( F_{A_{z}, B_{z}}(kt), F_{S_{z}, T_{z}}(t), F_{A_{z}, S_{z}}(t), F_{B_{z}, T_{z}}(kt) \right) \geq 0
\]

\[
\phi \left( F_{A_{z}, B_{z}}(kt), F_{A_{z}, B_{z}}(t), F_{A_{z}, A_{z}}(t), F_{B_{z}, B_{z}}(kt) \right) \geq 0
\]

\[
\phi \left( F_{A_{z}, B_{z}}(kt), F_{A_{z}, B_{z}}(t), 1, 1 \right) \geq 0
\]
as \( \phi \) is non-decreasing in first argument, we have

\[
\phi \left( F_{A_{z}, B_{z}}(t), F_{A_{z}, B_{z}}(t), 1, 1 \right) \geq 0
\]

using (ii), we have \( F_{A_{z}, B_{z}}(t) \geq 1 \) for all \( t > 0 \), which gives \( F_{A_{z}, B_{z}}(t) = 1 \). Thus \( A_{z} = B_{z} \) and hence \( A_{z} = B_{z} = z \). Combining all results, we get \( z = A_{z} = B_{z} = S_{z} = T_{z} \).

From this we conclude that \( z \) is a common fixed point of \( A, B, S \) and \( T \).

**Uniqueness.** Let \( z_{1} \) be another common fixed point of \( A, B, S \) and \( T \). Then \( z_{1} = A_{z_{1}} = B_{z_{1}} = S_{z_{1}} = T_{z_{1}} \) and \( z = A_{z} = B_{z} = S_{z} = T_{z} \) then by inequality (1), putting \( x = z \) and \( y = z_{1} \), we get

\[
\phi \left( F_{A_{z}, B_{z_{1}}}(kt), F_{S_{z}, T_{z_{1}}}(t), F_{A_{z}, S_{z}}(t), F_{B_{z_{1}}, T_{z_{1}}}(kt) \right) \geq 0
\]

\[
\phi \left( F_{z_{1}, z_{1}}(kt), F_{z, z_{1}}(t), F_{z, z_{1}}(t), F_{z_{1}, z_{1}}(kt) \right) \geq 0
\]

\[
\phi \left( F_{z, z_{1}}(kt), F_{z_{1}, z_{1}}(t), 1, 1 \right) \geq 0
\]
as \( \phi \) is non-decreasing in first argument, we have

\[
\phi \left( F_{z, z_{1}}(t), F_{z, z_{1}}(t), 1, 1 \right) \geq 0
\]

using (ii), we have \( F_{z, z_{1}}(t) \geq 1 \) for all \( t > 0 \), which gives \( F_{z, z_{1}}(t) = 1 \). Thus \( z = z_{1} \). Thus \( z \) is the unique common fixed point of \( A, B, S \) and \( T \).
If we increase the number of self maps from four to six then we have the following.

**Corollary 3.2** Let $A, B, S, T, I$ and $J$ be self mappings on a complete menger space $(X, F, t)$, satisfying

(a) $AB(X) \subseteq J(X)$ and $ST(X) \subseteq I(X)$,
(b) $(ST, J)$ is semi compatible,
(c) For some $\phi \in \Phi$, there exists $k \in (0,1)$ such that for all $x, y \in X$ and $t > 0$,

\[
\phi \left( F_{ABx,STy}(kt), F_{Ix,Jy}(t), F_{ABx,Ix}(t), F_{STy,Jy}(kt) \right) \geq 0 \quad (1)
\]
\[
\phi \left( F_{ABx,STy}(kt), F_{Ix,Jy}(t), F_{ABx,Ix}(kt), F_{STy,Jy}(t) \right) \geq 0 \quad (2)
\]

If the pair $(AB, I)$ and $(ST, J)$ share the common limit in the range of $I$ property, then $AB, ST, I$ and $J$ have a unique common fixed point. Furthermore, if the pairs $(A, B), (A, I), (B, I), (S, T), (S, J)$ and $(T, J)$ are commuting mapping then $A, B, S, T, I$ and $J$ have a unique common fixed point.

**Proof.** From theorem 3.1, $z$ is the unique common fixed point of $AB, ST, I$ and $J$. Finally, we need to show that $z$ is also a common fixed point of $A, B, S, T, I$, and $J$. For this, let $z$ be the unique common fixed point of both the pairs $(AB, I)$ and $(ST, J)$. Then, by using commutativity of the pair $(A, B), (A, I)$ and $(B, I)$, we obtain

\[ Az = A(ABz) = A(BAz) = AB(Az) , \quad Az = A(Iz) = I(Az), \]
\[ Bz = B(ABz) = B(A(Bz)) = BA(Bz) = AB(Bz) , \quad Bz = B(Iz) = I(Bz), \]

which shows that $Az$ and $Bz$ are common fixed point of $(AB, I)$, yielding thereby

\[ Az = z = Bz = Iz = ABz \quad (4) \]

in the view of uniqueness of the common fixed point of the pair $(AB, I)$. Similarly, using the commutativity of $(S, T), (S, J), (T, J)$, it can be shown that

\[ Sz = Tz = Jz = STz = z. \quad (5) \]

Now, we need to show that $Az = Sz(Bz = Tz)$ also remains a common fixed point of both the pairs $(AB, I)$ and $(ST, I)$. For this, put $x = z$ and $y = z$ in (1) and using (4) and (5), we get

\[ \phi \left( F_{ABz,STz}(kt), F_{Iz,Jz}(t), F_{ABz,Iz}(t), F_{STz,Jz}(kt) \right) \geq 0 \]

that is,
\[ \phi \left( F_{Az,Sz}(kt), F_{Az,Sz}(t), F_{Az,Az}(t), F_{Sz,Sz}(kt) \right) \geq 0 \]

as $\phi$ is nondecreasing in first argument, we have
\[ \phi \left( F_{Az,Sz}(t), F_{Az,Sz}(t), F_{Az,Az}(t), F_{Sz,Sz}(kt) \right) \geq 0 \]
\[ \phi \left( F_{Az,Sz}(kt), F_{Az,Sz}(t), 1, 1 \right) \geq 0 \]

using (ii), we obtain $F_{Az,Sz}(t) \geq 1$ for all $t > 0$, which gives $F_{Az,Sz}(t) = 1$, that is, $Az = Sz$. Similarly, it can be shown that $Bz = Tz$. Thus, $z$ is the unique common fixed point of $A, B, S, T, I,$ and $J$. 
References


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